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Phase diagram of neutron star quark matter in nonlocal chiral models

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Abstract. We analyze the phase diagram of two-flavor quark matter under neutron star constraints for a nonlocal covariant quark model within the mean-field approximation. Applications to cold compact stars are discussed.

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1 Introduction

The characteristics of the QCD phase diagram is presently an important open subject of research in particle physics. In particular, the behavior of strongly interacting matter in the region of low temperatures and large baryon densities could be tested against observational constraints from neutron stars [1]. Since in this thermodynamical region one finds strong difficulties when trying to perform lattice QCD calculations, the pictures emerging from different effective models of strong interactions deserve great interest from the theoretical point of view. Here we focus on a two-flavor chiral quark model which includes covariant nonlocal four-fermion interactions, motivated by an effective one-gluon exchange (OGE) picture. Nonlocality arises naturally in the context of several successful approaches to low-energy quark dynamics, such as the instanton liquid model [2] and the Schwinger-Dyson resummation techniques [3], and it is also a well-known feature of lattice QCD [4].

2 Formalism

The Euclidean action for the nonlocal model considered here, in the case of two light flavors and anti-triplet diquark interactions, is given by

$$S_E = \int \mathrm{d}^4 x \left\{ \bar\psi(x)\,(-i\partial\!\!\!/ + m)\,\psi(x) - \frac{G}{2} j^f_M(x) j^f_M(x) - \frac{H}{2}\left[j^a_D(x)\right]^\dagger j^a_D(x) \right\}. \qquad (1)$$

Here m is the current quark mass, which is assumed to be equal for u and d quarks, and $j_{M,D}$ are mesonic and diquark nonlocal currents. The nonlocality is introduced here in a covariant way, through a separable interaction arising from an effective OGE picture. In this way, we have

$$j^f_M(x) = \int \mathrm{d}^4 z\ g(z)\ \bar\psi\left(x+\frac{z}{2}\right)\ \Gamma_f\ \psi\left(x-\frac{z}{2}\right),$$
$$j^a_D(x) = \int \mathrm{d}^4 z\ g(z)\,\bar\psi_c\left(x+\frac{z}{2}\right)\ i\gamma_5\tau_2\lambda_a\ \psi\left(x-\frac{z}{2}\right), \qquad (2)$$

where $\psi_c(x) = \gamma_2\gamma_4\,\bar\psi^T(x)$, $\Gamma_f = (\mathbf{1}, i\gamma_5\boldsymbol{\tau})$, and $\boldsymbol{\tau}$ and λ_a, with $a = 2,5,7$, stand for Pauli and Gell-Mann matrices acting on flavor and color spaces, respectively. The function $g(z)$ is a form factor that characterizes the nonlocal interaction.

The effective action in eq. (1) might arise via Fierz rearrangement from some underlying more fundamental interactions, and is understood to be used —at the mean-field level— in the Hartree approximation. In general, the

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ratio of coupling constants H/G would be determined by these microscopic couplings; for example, OGE interactions lead to $H/G = 0.75$. Since the precise derivation of the effective couplings from QCD is not known, here we will leave H/G as a free parameter.

Standard bosonization of the theory leads, in the mean-field approximation, to a thermodynamical potential per unit volume given by

$$\Omega^{MFA} = \frac{\bar{\sigma}^2}{2G} + \frac{|\bar{\Delta}|^2}{2H} - \frac{T}{2} \sum_{n=-\infty}^{\infty} \int \frac{\mathrm{d}^3 \boldsymbol{p}}{(2\pi)^3} \ln \det \left[\frac{S^{-1}}{T} \right], \quad (3)$$

where the inverse propagator $S^{-1}(\bar{\sigma}, \bar{\Delta})$ is a 48×48 matrix in Dirac, flavor, color and Nambu-Gorkov spaces (its explicit form is given in ref. [5]). Here $\bar{\sigma}$ and $\bar{\Delta}$ stand for the mean-field values of scalar meson and diquark fields. Owing to the nonlocality, they come together with momentum-dependent form factors. The values of $\bar{\sigma}$ and $\bar{\Delta}$ can be obtained from the coupled gap equations

$$\frac{\mathrm{d}\Omega^{MFA}}{\mathrm{d}\bar{\Delta}} = 0\,, \qquad \frac{\mathrm{d}\Omega^{MFA}}{\mathrm{d}\bar{\sigma}} = 0\,. \quad (4)$$

In general one has to consider a different chemical potential μ_{fc} for each quark flavor f and color c. However, when the system is in chemical equilibrium, not all μ_{fc} are independent. In our case, it can be seen [5] that they can be written in terms of only three quantities: the baryon chemical potential μ_B, the quark electric chemical potential μ_Q and the color chemical potential μ_8. Defining $\mu = \mu_B/3$, the corresponding relations read

$$\begin{aligned} \mu_{qr} = \mu_{qg} &= \mu + Q_q\, \mu_Q + \mu_8/3\,, \\ \mu_{qb} &= \mu + Q_q\, \mu_Q - 2\,\mu_8/3\,, \end{aligned} \quad (5)$$

where, $q = u, d$, and Q_q are quark electric charges.

In the core of neutron stars, in addition to quark matter we have electrons. The latter can be thermodynamically treated as a free Fermi gas, and their contribution has to be added to the grand canonical thermodynamical potential. Moreover, quark matter and electrons have to be in β- equilibrium. Thus, assuming that antineutrinos escape from the stellar core, we must have

$$\mu_{dc} - \mu_{uc} = -\mu_Q = \mu_e\,. \quad (6)$$

If we now require the system to be electric and color charge neutral, the number of independent chemical potentials reduces further: μ_e and μ_8 are fixed by the conditions of vanishing electric and color densities. In this way, for each value of T and μ we should find the values of $\bar{\Delta}$, $\bar{\sigma}$, μ_e and μ_8 that solve eqs. (4), supplemented by the β-equilibrium and electric and charge neutrality conditions.

3 Numerical results

According to previous analyses carried out within nonlocal scenarios [6], the results are not expected to show a strong qualitative dependence on the shape of the form

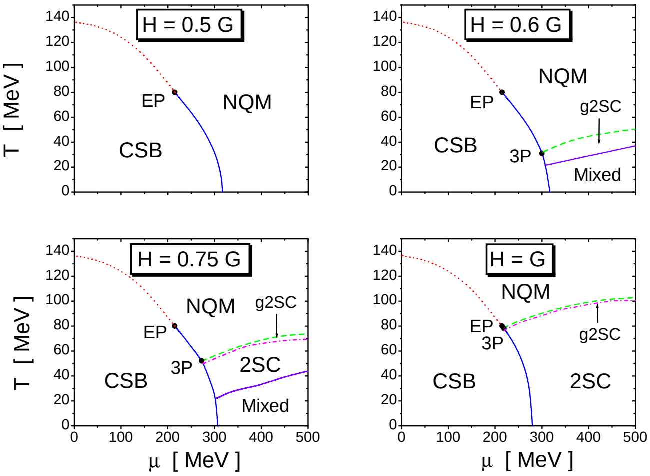

Fig. 1. Phase diagrams for the nonlocal OGE-based model, for different values of the ratio H/G, under neutron star constraints.

factor. Thus we will consider here a simple, well-behaved Gaussian function

$$g(p^2) = \exp(-p^2/\Lambda^2)\,, \quad (7)$$

where Λ is a free parameter, playing the rôle of an ultraviolet cut-off.

For definiteness, for the input parameters we choose $m = 5.12\,\mathrm{MeV}$, $\Lambda = 827\,\mathrm{MeV}$ and $G\Lambda^2 = 18.78$. These values are fixed so as to reproduce the empirical values for the pion mass m_π and decay constant f_π, and lead to a phenomenologically reasonable value for the chiral condensate [7] at vanishing T and μ_B, namely $\langle 0|\bar{q}q|0\rangle^{1/3} = -250\,\mathrm{MeV}$. The only remaining free parameter is the coupling strength H in the scalar diquark channel. We choose here values for H/G in the range from 0.5 to 1, *i.e.* around the Fierz value $H/G = 0.75$ discussed above.

Our results for the phase diagrams are shown in fig. 1, where we plot the phase transition curves on T-μ diagrams for different ratios H/G. In the graphs we show the regions corresponding to different phases, as well as the position of triple points (3P) and end points (EP). Besides the region of low T and μ, in which the chiral symmetry is broken (CSB), one finds normal quark matter (NQM) and two-flavor superconducting (2SC) phases. Between CSB and NQM phases one has first order and crossover transitions —represented by solid and dotted lines, respectively—, whereas between NQM and 2SC regions, in all cases, we find a second order phase transition —dashed lines in the diagrams of fig. 1. Close to this NQM/2SC phase border, the dashed-dotted lines in the graphs delimit a band that corresponds to the so-called gapless 2SC (g2SC) phase. In this region, in addition to the two gapless modes corresponding to the unpaired blue quarks, the presence of flavor asymmetric chemical potentials gives rise to another two gapless fermionic quasiparticles. For the range of parameters considered here, however, the g2SC region is too narrow to lead to sizeable effects. Finally, for intermediate values of H/G we find a region in which a certain volume fraction of the quark matter undergoes a transition to the 2SC phase coexisting with the remaining NQM phase. This is a 2SC-NQM mixed phase in which the system realizes the constraint of electric neutrality globally: the coexisting phases have opposite electric charges which neutralize each other, at a common equilibrium pressure.

It can be seen that the 2SC phase region becomes larger when the ratio H/G is increased. This is not surprising, since H is the effective coupling governing the quark-quark interaction that gives rise to the pairing. As a general conclusion, it can be stated that even under neutron star constraints, provided the ratio H/G is not too low, the nonlocal scheme favors the existence of color superconducting phases at low temperatures and moderate chemical potentials (we do not find 2SC only for $H/G = 0.5$). This is in contrast to the situation in the NJL model [8], where the existence of a 2SC phase turns out to be rather dependent on the input parameters. Our results are also qualitatively different from those obtained in the case of noncovariant nonlocal models [9], where above the chiral phase transition the NQM phase is preferable for values of the coupling ratio $H/G \lesssim 0.75$, and a color superconducting phase can be found only for $H/G \approx 1$. It is now desirable to extend the present studies to other pairing channels relevant for neutron star cooling, such as spin-1 pairing, where at present only the NJL [10] and noncovariant nonlocal models [11] have been considered.

4 Application to cold compact stars

One of the most important present applications of the microscopic approaches to quark matter is to study whether such a state of matter can exist in the interior of cold compact stars. Let us briefly discuss our ongoing investigations on this issue. Unfortunately, a consistent relativistic approach to the quark-hadron phase transition, where hadrons appear as bound states of quarks, has not been developed up to now. Thus, we apply a so-called two-phase description, in which the nuclear matter phase is described within the relativistic Dirac-Brueckner-Hartree-Fock (DBHF) approach (see, *e.g.*, ref. [12]) and the transition to a quark matter phase is obtained by a Maxwell construction. From the curves presented in the previous section, it can be seen that in our OGE-inspired nonlocal model one finds a relatively low value of the critical density at $T = 0$, hence some extra repulsion is needed in order to obtain a more realistic value. We have found that this can be achieved by including some small additional interaction in the omega vector meson channel, which does not affect in general the qualitative features of the phase diagrams discussed in the previous section. Given the corresponding equation of state, the mass and structure of spherical, nonrotating stars is obtained by solving the Tolman-Oppenheimer-Volkov equations. Our preliminary results confirm the findings of ref. [13], *i.e.* that compact stars with quark matter cores are consistent with modern observations. Moreover, for the nonlocal quark models described here, this is found to be possible for values of H/G closer to the standard, OGE motivated ratio $H/G = 0.75$.

5 Conclusions

We have studied the phase diagram of two-flavor quark matter under neutron star constraints for a nonlocal, covariant quark model within the mean field approximation. The form of the nonlocal coupling has been motivated by a separable approximation of the OGE interaction. We have considered a nonlocal form factor of a Gaussian shape, and the model parameters (current quark mass m, coupling strength G, UV cutoff Λ) have been fixed so as to obtain adequate values for the pion mass, the pion decay constant and the chiral condensate at vanishing T and μ_B.

After the numerical evaluation of the gap equations at finite temperature and chemical potential, considering different values for the coupling strength in the scalar diquark channel, we have found that different low-temperature quark matter phases can occur at intermediate densities: normal quark matter (NQM), pure superconducting (2SC) quark matter and mixed 2SC-NQM phases. A band of gapless 2SC phase appears at the border of the superconducting region, but this occurs in general at nonzero temperatures and should not represent a robust feature for compact star applications. Finally, in the context of the nonlocal theory discussed here, we have obtained preliminary results showing that compact stars with quark matter cores turn out to be consistent with modern observations.

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